Midterm review session

CS 103 CAs

Summer

25 July 2023

What will we cover?

- Lecture 0 to 2: Proofs and Set Theory
- Lecture 3 to 5: First-order Logic
- Lecture 6 to 8: Functions and Graphs

Proofs and Set Theory

Let's talk about sets!

Proofs and Set Theory

How do we approach set theory problems?

Remark

Our primary problem: Does $A \subseteq B$?
Our primary approach: $\forall a \in A$, prove that $a \in B$.

Many, many, many problems take this form.

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Let's practice!

Proofs and Set Theory

Let A and B be arbitrary sets. Prove that $\wp(A) \cap \wp(B) = \wp(A) - \wp(B)$.

What is an element $X \in \wp(A) \cap \wp(B)$?

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$X \in \wp(A)$ but $X \notin \wp(B)$!

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What about an element in $X \in \wp(A) - \wp(B)$?

$X \in \wp(A)$ but $X \notin \wp(B)$!

Wait a second. Does this make sense? Nope.

How can an element be in a set ($X \in \wp(B)$) and not in a set ($X \notin \wp(B)$)?

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Let A and B be arbitrary sets. Prove that $\wp(A) \cap \wp(B) \subseteq \wp(A \cap B)$.

What is an element $X \in \wp(A) \cap \wp(B)$?

Well, an element $X \in \wp(A)$ is a subset of A i.e. $X \subseteq A$.

What about an element in $X \in \wp(A) \cap \wp(B)$? That's like, $X \in \wp(A)$ and $X \in \wp(B)$.

X must be a subset of both A and B.

Remark

Now, we know that for X in $\wp(A) \cap \wp(B)$, $X \subseteq A$ and $X \subseteq B$.

Let's work on $\wp(A \cap B)$.

Proofs and Set Theory

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What is an element $X \in \wp(A) \cap \wp(B)$?

Well, an element $X \in \wp(A)$ is a subset of A i.e. $X \subseteq A$.

What about an element in $Y \in \wp(A) \cap \wp(B)$? That's like, $Y \in \wp(A)$ and $X \in \wp(B)$.

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What is an element $Y \in \wp(A \cap B)$?

By definition, Y must be a subset of $A \cap B$.

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X must be a subset of both A and B.

What is an element $Y \in \wp(A \cap B)$?

By definition, Y must be a subset of $A \cap B$.

Thus, $\forall y \in Y$, $y \in A$ and $y \in B$.

Therefore, Y is a subset of both A and B

Proofs and Set Theory

Wow! Wasn't that fun?

Proofs and Set Theory

Wow! Wasn't that fun?
Fortunately, we're not done yet.

We have to actually write a proof.

Remark

For practice, we won't work through the proof here.

Trust us, it's better that you do this on your own. However! The solution can be found on the website, under Exams → Extra Practice Problems 1. This was #20.

Remark

#21 is a good practice problem too.

Proofs and Set Theory

Let's talk about proofs and set theory again!

Practice midterm 2 probblem 3: modular congruence of squares

Definition

Recall from Problem Set 1 the following definition for $\equiv_k$: for any integers a, b, and k when there exists an integer q such that

$$a = b + qk$$

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Proofs and Set Theory

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2. For all integers x, y and k, if $x \equiv_k y$, then $x^2 \equiv_k y^2$.

1. Statement 1: For all integers x, y and k, if $x^2 \equiv_k y^2$, then $x \equiv_k y$.

Step 1: determining if statement 1 is true or false

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- Thought 3: The numbers $\pm a$ differ by $2a$, so unless $k = 2a$, these should be different modulo k.
- Thought 4: Let's find numbers like this! Take the numbers $+1$ and -1. We want k to not be 2, so let $k = 3$. Then -1 and 1 are not congruent mod 3 as desired. But $1^2 = (-1)^2 = 1$.

Proofs and Set Theory

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Now what would you actually write? Something like this!

This statement is false. Pick $x = 1$ and $y = -1$, and $k = 3$. Then $x^2 = 1$ and $y^2 = 1$, so $x^2 \equiv_k y^2$. However, $x \not\equiv_k y$ since there is no choice of q such that $1 = -1 + 3q$.

Proofs and Set Theory

Statement 2: For all integers x, y and k, if $x \equiv_k y$, then $x^2 \equiv_k y^2$.

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- Thought 3: Scratch work!

$$
\begin{aligned}
x &= y + qk \\
x^2 &= (y + qk)^2 \\
&= y^2 + 2yqk + q^2k^2
\end{aligned}
$$

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$$\begin{aligned} x &= y + qk \\ x^2 &= (y + qk)^2 \\ &= y^2 + 2yqk + q^2k^2 \end{aligned}$$

- Thought 4: Oh, if you factor out a k this aligns with the definition of $x^2 \equiv_k y^2$.

$$x^2 = y^2 + (2yq + q^2k)k$$

Proofs and Set Theory

Now that we have done this scratch work, we are ready to write a proof. What would we actually write? Time for the proof-writing checklist!

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2. State want-to-shows (we want to show there exists p such that $x^2 = y^2 + pk$)

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4. Show this value works!

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3. Select the value of p we found out before
4. Show this value works!

Try writing this proof yourself and then check the solutions for a complete proof!

First-order Logic

Let's talk about logic!

First-order Logic

Remark

Why do we care about logic?

To us, mathematical logic is important because it forms the foundation of the theory of computation:

"What are the fundamental capabilities and limitations of computers?"

Also, Alan Turing was a logician, so... we're in good company.

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With that out of the way, let's translate some sentences!

Extra Practice Problem #5: Before the Law

Here's our sentence:

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

And here our predicates:

- $Person(p)$
- $Gate(g)$
- $MayPass(p, g)$
- $WillPass(p, g)$

Let's focus on our sentence.

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

What should we look for?

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

What should we look for?

Let's simplify.

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

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Let's simplify.

Pause! Think about this for yourself, especially if you can pause.

(We don't have that luxury, unfortunately.)

How would **you** break down this sentence?

First-order Logic

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A note from your TA:

We apologize to all non-native English speakers.

If it's helpful, I would note that each highlighted section is "its own sentence"

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

Some person has a gate = There is a person p, and there is a gate g

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

$$\begin{aligned}\text{Some person has a gate} &= \text{There is a person } p\text{, and there is a gate } g \\ &= \exists p\,(Person(p) \wedge \exists g\,(Gate(g) \wedge \ldots)) \\ &= \exists p\,(Person(p) \wedge \exists g\,(Gate(g) \wedge \text{A} \wedge \text{B}))\end{aligned}$$

First-order Logic

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$$\exists p\,(Person(p) \land \exists g\,(Gate(g) \land \text{A} \land \text{B})$$

$$\begin{aligned}\text{They alone may pass...} &= \text{A} \\ &= \text{They alone} \rightarrow MayPass(p, g)\end{aligned}$$

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And then we plug that all back in!

First-order Logic

Some person has a gate that they alone are permitted to pass through, but which they will never pass through

$$\exists p\,(Person(p) \;\wedge \exists g\,(Gate(g) \wedge$$
$$(\forall q\,(Person(q) \wedge p \neq q \to \neg MayPass(q, g))$$
$$\wedge\; \mathrm{B}))$$

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& \wedge \neg WillPass(p, g) \\
&)))
\end{aligned}
$$

And we're done! At last!

Functions and Graphs

Let's talk about functions and graphs!

Functions and Graphs

Practice exam 2 problem 2

Let $G = (V, E)$ be an undirected graph. We define the **adjacency function** for G as the function $f_A : V \to \wp(V)$, where $f_A(u) = \{v \in V | u, v \in E\}$.
Let $G = (V, E)$ be an undirected graph. We define the **reachability function** for G as the function $f_R : V \to \wp(V)$, where $f_R(u) = \{$ there is a path from u to $v\}$.

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- Question 1: Define a graph $G = (V, E)$, for which there exists a $v \in V$ where $f_A(v) = \emptyset$ (where f_A is the graph's adjacency function), or state that no such graph exists.

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- Thought 3: Does this really work? Yes!

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- Thought 2: What is the simplest graph which has a vertex with no path to any vertex? What about the same example as before, $V = \{1\}$ and $E = \{\}$? (be suspicious)

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- Thought 3: Does this really work? No! Since there is a path from v to itself.
- Thought 4: No graph exists.

Functions and Graphs

Consider the graph $G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}, \{5, 6\}\}$

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- Question 3: What is $f_A(1)$?

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- Question 3: What is $f_A(1)$? We can just look at edges containing 1! This will be 2 and 4.
- Question 4: What is $f_R(1)$?

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- Question 3: What is $f_A(1)$? We can just look at edges containing 1! This will be 2 and 4.
- Question 4: What is $f_R(1)$? We might want to draw a picture...

Functions and Graphs

Picture:

Functions and Graphs

Picture:

Sanity check: Does this picture have all the edges and vertices in the graph? $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}, \{5, 6\}\}$

Picture:

Sanity check: Does this picture have all the edges and vertices in the graph? $V = \{1, 2, 3, 4, 5, 6\}$ and $E = \{\{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 1\}, \{5, 6\}\}$
Now that we have the picture: $f_R(1) = 1, 2, 3, 4$